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LIBRARY AIDS TO MATHEMATICAL STUDY.

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Herr Valentin of Berlin who has been working on a general mathematical bibliography for more than twenty years estimates that the total number of different mathematical works is about 35,000 and that about 95,000 mathematical articles have appeared in the various periodicals.* Morever, the amount of this literature is growing at an increasing rate of speed so that it appears likely that during the next forty years there will be a larger addition to the mathematical literature than the total amount which has appeared up to the present time. In fact, this is a very conservative estimate, since such a work as the *Jahrbuch der Fortschritte der Mathematik* chronicles annually about 2000 books and articles in pure mathematics in addition to a large number in closely allied subjects.

One of the first questions which confronts the student is the relative importance of periodic and non-periodic publications. In general it must be said that these supplement each other and that the advanced student needs both. As the books generally co-ordinate the results obtained by many different writers, broad views can usually be more easily obtained from books than from the separate articles, but these broad views are tinged by the peculiar bent of the author's mind and they naturally do not exhibit the clearness in detail which the student would have obtained by studying the authorities himself. This is especially true of the newer subjects where the number of books is comparatively small and where progress is generally so rapid that the books are deplorably behind the times.

The history of mathematics furnishes a good illustration of the point in question. When the first edition of the first three volumes of Cantor's *Vorlesungen über Geschichte der Mathematik* appeared, it was commonly regarded as authoritative even in nearly all of the details. It was to a large extent instrumental in arousing a more general interest in the history of mathematics and marked the beginning of an unusually active period in historical investigations, so that it is becoming much more difficult for one man

*Felix Mueller, *Bibliotheca Mathematica*, Vol. 7 (1907), p. 416.

to make use of all the available knowledge for a general history. Hence we find that the first volume of the third edition of this great work, which has appeared recently, and contains a large number of improvements over the preceding editions, has met with severe criticism,* and it appears that the student of mathematical history is compelled to get his knowledge largely from the journals if he wants to feel certain that he is not holding views which have been proved to be incorrect in well known literature.

Ever since mathematics has had a considerable literature both the investigator and the student have felt the need of better facilities to learn just what has been done. A very interesting discussion of this need is found among the earliest publications of the Royal Society of London. In the volume for 1681-2 Dr. Pell suggests "that the three following works be composed and published: 1. *Mathematical Pandects*, containing, as clearly, methodically, concisely, and ingeniously, as can be done, whatever may be collected, or deduced by way of corollary, from mathematical books and discoveries made before our time; quoting the most ancient authors in which they are found, and noting in all following authors where they have pilfered from others without acknowledgment; or, what is worse, have arrogated to themselves the inventions of others. By this means, that large library would be contracted into a narrower compass, to the great saving of labour, time, and expence, for those that come after. 2. *A Mathematical Compendium*, containing, in a concise manual, all the most useful tables, with precepts to show their application to the solution of problems, either of pure mathematics, or applied to other subjects. Finally, that we may not always be confined to books in this kind of learning, there should be contrived, 3. *The Self-sufficient Mathematician*, or an instruction to show how any mathematician, not averse to labour, may acquire so much skill, that without the aid of books or instruments he may accomplish the solution of any mathematical problem, and that as easily as another by only turning over books."

In answering some suggestions by the noted French mathematician, Mersenne, Dr. Pell makes the following interesting remarks in the same volume:† "Now the less I am pleased with these minute mathematicians, the more I should wish for a library of this kind, as being the only method of curing that licentious itch of scribbling. For these prating pretenders, ever trifing in a childish manner, while they would seem to accommodate themselves to the capacity of youth, may see that there are already too many who have compiled rudiments of this kind. And those who fondly aim at advancing the mathematical sciences by an infinity of new discoveries, when they see so many empty paradoxes, which have been condemned and ridiculed by the public, may take warning by the miscarriage of others. But especially the plagiaries, those pests of all true literature, will not have the

*Ref. G. Enestroem, *Bibliotheca Mathematica*, Vol. 7 (1907), p. 398.

†These extracts are interesting on account of the style and are illustrations of some of the earliest periodic mathematical literature.

impudence to vend, as their own, any old books, or any parts of them, which perhaps have not been printed more than once."

While these statements, coming to us through more than two centuries, do not fit to present conditions yet they exhibit the same yearning for convenient means to learn the known along some lines. Fortunately, an increasing number of mathematicians have been willing to devote their best energies to the work of making it easier for others to find out what is known about a particular subject. Although our best bibliographical works are far from perfect yet they are of an incalculable value for the advancement of knowledge. Some of these, like the *Juhrbuch der Fortschritte der Mathematik,* have been conducted by one or two men with such assistance as they could get from their colleagues; others, like the *Revue semestrielle des publications mathêmatiques* and the *Royal Society Catalogue,* are conducted by bodies of learned men; still others, like the *International catalogue of scientific literature,* are directed by international councils and supported by a large number of different countries.

One of the greatest advantages of such bibliographical works is that they enable the student to find quickly what has been done along a particular line. A classification which has been very extensively adopted is due to the international congress of mathematical bibliography held at Paris in 1889. It aims to give a very detailed classification of mathematical subjects but *does not consider the different methods employed in treating these subjects.* This classification is explained in a small volume entitled *Index du rêpertoire bibliographique des sciences mathêmatiques* published by Gauthier-Villars et Fils of Paris. Among many other places it has been adopted by the *Revue semestrielle* and by the *Bulletin of the American Mathematical Society.* The largest divisions of the entire subject of mathematics are denoted by capital letters of the Roman alphabet, subdivisions when necessary being denoted by exponents. Further successive subdivisions are indicated by number symbols, small Roman letters and small Greek letters. Hence this scheme provides for an almost endless division and even at the present time it sometimes enables one to obtain all the classified literature on a particular subject by looking over less than one thousandth part of the entire mathematical literature of the period.

From the above it is evident that the books on books are almost as important to the student as the original works themselves. In fact, most advanced students will probably use these books on books more frequently than any other equal number of volumes. In addition to the four great bibliographical works which have been mentioned the great encyclopedias (German and French) which are now in the process of publication, and the *Encyklopaedie der Elementar-Mathematik* by Weber and others, which was completed recently, are especially helpful to the student. Hagen's *Synopsis der hoeheren Mathematik* and Carr's *Synopsis of elementary results of pure mathematics* are also frequently very convenient. It is however not our

purpose to give a long list of bibliographical aids to the mathematical student. Such a list may be found in *Jahresbericht der Deutschen Mathematiker-Vereinigung,* volume 12 (1903), pp. 408-426. Our main object has been to convey an accurate idea in reference to the magnitude of the total mathematical literature and some of the aids to use this literature wisely in the better libraries.

THREE THEOREMS ON THE TRISECTION OF AN ACUTE ANGLE.

By J. SAMSONOFF, New York City.

THEOREM I. *The line DE (Fig. 1), which passes through the vertex A of one of the equal angles of an isosceles triangle ACB and intercepts on the line BC a part DC equal to the chord EC subtending the arc EC, which is drawn with radius AC from point A as a center, is the trisectorial line for the angle ABC.*

We have given the isosceles triangle ACB and the circumference FEC, which is drawn from the point A as a center and with AC as a radius, also the line DE which passes through A and intercepts on BC a part DC equal to the chord EC.

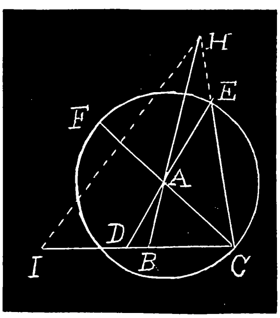

Fig. 1.

We are to prove that the line DE is the trisectorial line for $\angle ABC$, that is, $\angle DAD = \frac{\angle ABC}{3}$.

PROOF. Prolong the line CE until it intersects the line BA at the point H, and from H draw a parallel to ED, which will intersect the line DC at the point I.

Now, $\angle CAB$, the exterior of triangle HCA, is equal to $\angle ACH + \angle CHA$; $\angle ABC$, the exterior angle of triangle IHB, is equal to $\angle BIH + \angle IHB$; but triangle EDC is isosceles (by hypothesis), triangle HCI is isosceles (by construction, as HI is parallel to ED); and triangle CAE is isosceles (because $AC = AE$).

Therefore, $\angle ACH = \angle BIH$, and $\angle IHB = \angle CHA$. But $\angle IHB = \angle HAE$ (lines IH and DE are parallel by construction).

Therefore triangle AEH is an isosceles triangle and $AE = EH$. Hence, $\angle ABC = \angle DAB + \angle ADB = 3(\angle DAB)$, or line DE is the trisectorial line for $\angle ABC$.

THEOREM II. *The bisector CF of the angle at the vertex of an isosceles triangle ACB (Fig. 2), prolonged to the intersection with the trisectorial line DE, forms an isosceles triangle FEC.*